

Solution EADM Workshop Multilevel Regression

Reference

Schultze, T., & Rakotoarisoa, A. F. (2015). Effects of distance between initial estimates and advice on advice utilization. *Judgment and Decision Making*, 10(2), 144-171.

Details Study 1

Task

60 Trials: Airline distance between European Union capitals

3 Phases: Initial Estimate (IE), Advise Estimate (AE), Final Estimate (FE)

Independent Variable

Advise Estimate: -75% to 75% (in steps of 5%) from IE

Example

IE: 600km, AD (10%) 660km, FE 630km

Dependent Variable

$$AT = \frac{IE - FE}{IE - AD} = .50$$

Range of score: restricted between 0 to 1

Read in the data

```
dataStudy1 = read.csv(file="http://journal.sjdm.org/14/141112a/ferni1.csv")
head(dataStudy1)
```

```
##      X VP trial dev      AT DC  IE  FE  AD abs_dev abs_dev_sq log_dev
## 1 1.1  1    1  10 0.0000000  0  500  500  550      10      100 2.302585
## 2 2.1  2    1  10 0.0000000  3 1000 1000 1100      10      100 2.302585
## 3 3.1  3    1  10 0.0000000  0  700  680  770      10      100 2.302585
## 4 4.1  4    1  10 1.0000000  1 3000 3700 3300      10      100 2.302585
## 5 5.1  5    1  10 0.3333333  1  600  620  660      10      100 2.302585
## 6 6.1  6    1  10 0.7142857  0  700  750  770      10      100 2.302585
##      shift ignore
## 1 0.0000000      1
## 2 0.0000000      1
## 3 0.0000000      1
## 4 0.2333333      0
## 5 0.0333333      0
## 6 0.07142857     0
```

Install and Load Libraries

```
# install.packages("arm")
# install.packages("lattice")
# install.packages("lmerTest")
```

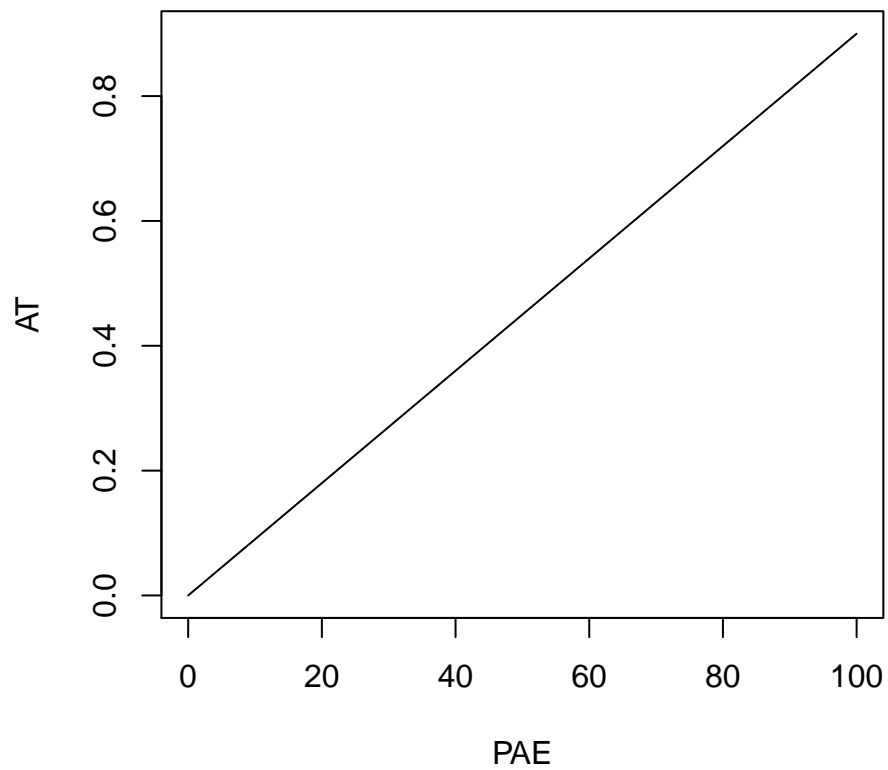
```
library("arm")
library("lattice")
library("lmerTest")
```

Three Proposed Models

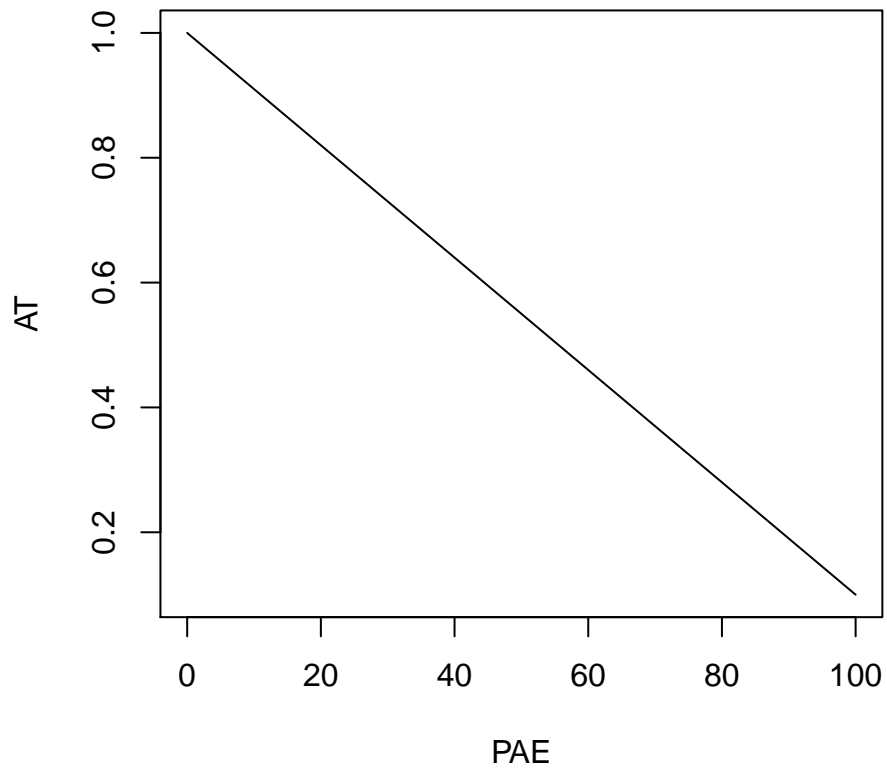
Linear model

$$AT = (\beta_{00} + u_{0j}) + (\beta_{01} + u_{1j}) \times PAE_{ij} + e_{ij}$$

```
curve(0+.009*x,0,100,xlab="PAE",ylab="AT")
```



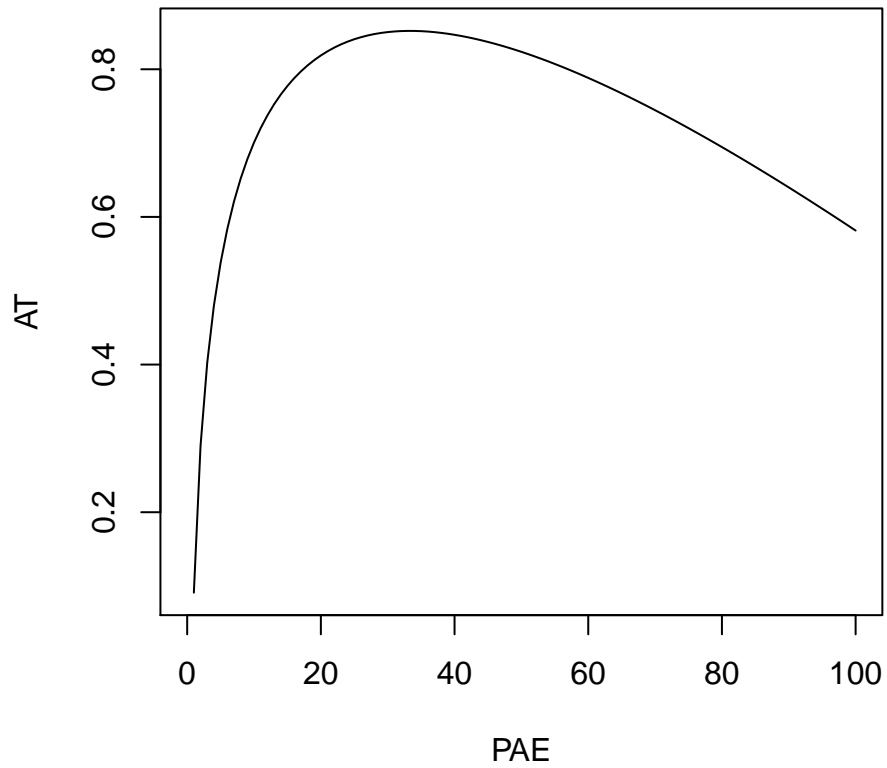
```
curve(1-.009*x,0,100,xlab="PAE",ylab="AT")
```



Log-model

$$AT = (\beta_{00} + u_{0j}) + (\beta_{01} + u_{1j}) \times PAE_{ij} + (\beta_{02} + u_{2j}) \times \log(PAE_{ij}) + e_{ij}$$

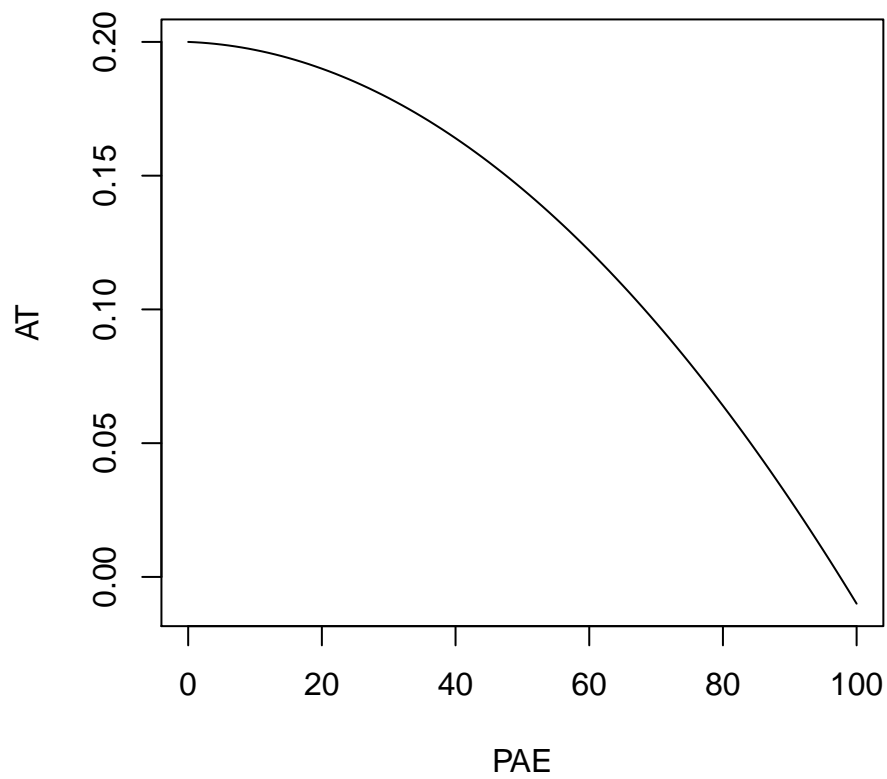
```
curve(.1-.009*x+.3*log(x),0,100,xlab="PAE",ylab="AT")
```



Squared-model

$$AT = (\beta_{00} + u_{0j}) + (\beta_{01} + u_{1j}) \times PAE_{ij} + (\beta_{02} + u_{2j}) \times (PAE_{ij})^2 + e_{ij}$$

```
curve( .2+ -.0001*x+ -.00002*(x)^2,0,100,xlab="PAE",ylab="AT")
```



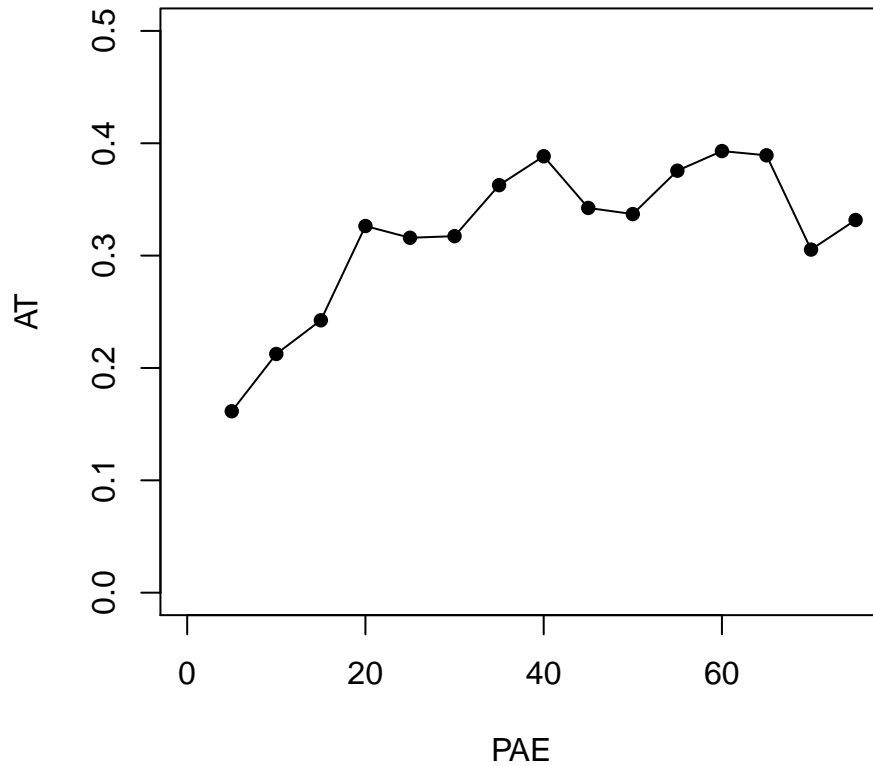
Plot Data

```
stepsManip = seq(0,75,5)

totalMeans = numeric()

for(loop in stepsManip){
  totalMeans = c(totalMeans,mean(dataStudy1$AT[dataStudy1$abs_dev==loop]))
}

plot(stepsManip,totalMeans,pch=16,xlab="PAE",ylab="AT",ylim=c(0,.5))
points(stepsManip,totalMeans,type="l")
```



Run Multilevel Models

All models contain a random intercept and random slopes for all predictors.

Fit Linear Model

$$AT = (\beta_{00} + u_{0j}) + (\beta_{01} + u_{1j}) \times PAE_{ij} + e_{ij}$$

```
linearMod = lmer(AT ~ 1 + abs_dev + (1 + abs_dev | VP),
  data = dataStudy1, REML = FALSE)
summary(linearMod)
```

```
## Linear mixed model fit by maximum likelihood t-tests use Satterthwaite
## approximations to degrees of freedom [lmerMod]
## Formula: AT ~ 1 + abs_dev + (1 + abs_dev | VP)
## Data: dataStudy1
##
##      AIC      BIC   logLik deviance df.resid
##  48.7    80.6   -18.3    36.7    1505
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
```

```

## -2.9857 -0.5888 -0.1293  0.6236  3.9321
##
## Random effects:
##   Groups   Name                Variance Std.Dev. Corr
##   VP       (Intercept) 3.065e-02 0.175067
##           abs_dev      4.108e-06 0.002027 0.05
##   Residual                    5.523e-02 0.235020
## Number of obs: 1511, groups:  VP, 27
##
## Fixed effects:
##           Estimate Std. Error      df t value Pr(>|t|)
## (Intercept) 2.408e-01  3.602e-02 2.710e+01  6.687 3.47e-07 ***
## abs_dev     2.049e-03  4.808e-04 2.724e+01  4.262 0.000217 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##           (Intr)
## abs_dev -0.147

```

Fit Log-Model

$$AT = (\beta_{00} + u_{0j}) + (\beta_{01} + u_{1j}) \times PAE_{ij} + (\beta_{02} + u_{2j}) \times 10 \times \log(PAE_{ij}) + e_{ij}$$

```

log_devSc = dataStudy1$log_dev * 10

logMod = lmer(AT ~ 1 + abs_dev + log_devSc + (1 + abs_dev + log_devSc|VP),
              data = dataStudy1, REML = FALSE)
summary(logMod)

```

```

## Linear mixed model fit by maximum likelihood t-tests use Satterthwaite
## approximations to degrees of freedom [lmerMod]
## Formula: AT ~ 1 + abs_dev + log_devSc + (1 + abs_dev + log_devSc | VP)
## Data: dataStudy1
##
##      AIC      BIC    logLik deviance df.resid
##    -4.6    48.6    12.3    -24.6    1501
##
## Scaled residuals:
##      Min      1Q  Median      3Q      Max
## -3.0785 -0.5888 -0.1595  0.5470  4.0821
##
## Random effects:
##   Groups   Name                Variance Std.Dev. Corr
##   VP       (Intercept) 1.145e-01 0.338431
##           abs_dev      2.429e-05 0.004928 0.79
##           log_devSc    2.493e-04 0.015789 -0.87 -0.92
##   Residual                    5.210e-02 0.228261
## Number of obs: 1511, groups:  VP, 27
##
## Fixed effects:
##           Estimate Std. Error      df t value Pr(>|t|)
## (Intercept) -0.094774  0.083086 26.699000  -1.141 0.264130

```

```
## abs_dev      -0.002963   0.001239  26.156000  -2.392 0.024238 *
## log_devSc    0.015426   0.003813  26.167000   4.046 0.000411 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##           (Intr) abs_dv
## abs_dev    0.809
## log_devSc -0.906 -0.927
```

Fit Squared Model

$$AT = (\beta_{00} + u_{0j}) + (\beta_{01} + u_{1j}) \times PAE_{ij} + (\beta_{02} + u_{2j}) \times \frac{1}{100} \times (PAE_{ij})^2 + e_{ij}$$

```
abs_dev_sqResc = dataStudy1$abs_dev_sq/100
sqMod = lmer(AT ~ 1 + abs_dev + abs_dev_sqResc + (1 + abs_dev + abs_dev_sqResc|VP),
             data = dataStudy1, REML = FALSE)
summary(sqMod)
```

```
## Linear mixed model fit by maximum likelihood t-tests use Satterthwaite
## approximations to degrees of freedom [lmerMod]
## Formula:
## AT ~ 1 + abs_dev + abs_dev_sqResc + (1 + abs_dev + abs_dev_sqResc |
## VP)
## Data: dataStudy1
##
##      AIC      BIC    logLik deviance df.resid
##      0.6      53.8      9.7    -19.4     1501
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.1699 -0.5890 -0.1651  0.5680  4.0980
##
## Random effects:
## Groups   Name                Variance Std.Dev. Corr
## VP      (Intercept)          3.164e-02 0.177877
##          abs_dev             6.266e-05 0.007916 -0.31
##          abs_dev_sqResc      7.210e-05 0.008491  0.30 -0.97
## Residual                    5.249e-02 0.229111
## Number of obs: 1511, groups:  VP, 27
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)   0.130335   0.040044  27.582000   3.255 0.002998 **
## abs_dev       0.009779   0.001938  27.010000   5.047 2.68e-05 ***
## abs_dev_sqResc -0.009631   0.002186  26.883000  -4.407 0.000151 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##           (Intr) abs_dv
```



```
## abs_dev      -0.497
## abs_dv_sqRs  0.468 -0.972
```

Compare Models Using the Likelihood Ratio Test

Test Linear versus Log Model

```
anova(linearMod,logMod)
```

```
## Data: dataStudy1
## Models:
## object: AT ~ 1 + abs_dev + (1 + abs_dev | VP)
## ..1: AT ~ 1 + abs_dev + log_devSc + (1 + abs_dev + log_devSc | VP)
##      Df    AIC    BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## object 6 48.658 80.581 -18.329  36.658
## ..1   10 -4.562 48.643  12.281 -24.562 61.22    4 1.607e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Test Linear versus Squared Model

```
anova(linearMod,sqMod)
```

```
## Data: dataStudy1
## Models:
## object: AT ~ 1 + abs_dev + (1 + abs_dev | VP)
## ..1: AT ~ 1 + abs_dev + abs_dev_sqResc + (1 + abs_dev + abs_dev_sqResc |
## ..1:    VP)
##      Df    AIC    BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## object 6 48.658 80.581 -18.3288  36.658
## ..1   10  0.586 53.791  9.7071 -19.414 56.072    4 1.937e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Test Log versus Squared Model

Bayesian information criterion.

```
BICs = BIC(logMod,sqMod)
```

```
BICs = data.frame(BICs)
```

```
BICs
```

```
##      df    BIC
## logMod 10 48.64294
## sqMod  10 53.79102
```

Bayes factor

```
exp((BICs[2,2] - BICs[1,2])/2)
```

```
## [1] 13.11874
```

Bonus Task

Test for the Log-Model if we need all random effects (Hint: nested models; test starts with L).

Print and Plot Random Effects

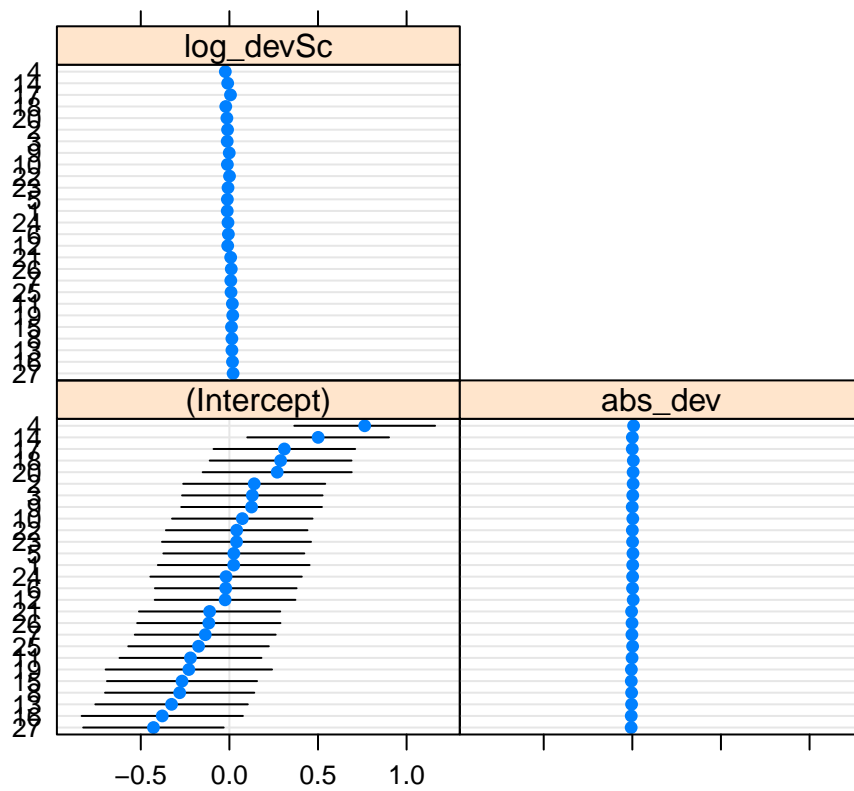
```
ranef(logMod)
```

```
## $VP
##      (Intercept)      abs_dev      log_devSc
## 1  0.02444579  0.0022032535 -0.0121002892
## 2  0.14013174  0.0045494865 -0.0102646530
## 3  0.12938669  0.0024550710 -0.0122413846
## 4  0.76346104  0.0071389043 -0.0229845955
## 5  0.02502868  0.0034267500 -0.0114220460
## 6 -0.02051552  0.0011046381 -0.0056433301
## 7 -0.13626241 -0.0023907587  0.0080050245
## 8 -0.28059357 -0.0039641636  0.0140944888
## 9  0.12516583 -0.0005988885 -0.0008109228
## 10 0.07285746  0.0025467721 -0.0113313189
## 11 -0.21958611 -0.0014808515  0.0169881599
## 12 -0.02431117  0.0048771166 -0.0096427238
## 13 -0.32630168 -0.0043161016  0.0141704711
## 14  0.50076592  0.0001663427 -0.0096312745
## 15 -0.26737378 -0.0057993109  0.0117668654
## 16 -0.37878783 -0.0061223484  0.0175377788
## 17  0.31013401 -0.0004429105  0.0062670763
## 18  0.28904421  0.0056165946 -0.0204093811
## 19 -0.22877269 -0.0053480723  0.0188565949
## 20  0.26971843  0.0043812687 -0.0140829491
## 21 -0.11164453 -0.0041289801  0.0072157646
## 22  0.04122889  0.0000155346  0.0006686806
## 23  0.04022186  0.0013244984 -0.0078229147
## 24 -0.01871451  0.0015998654 -0.0078883293
## 25 -0.17406334  0.0015259701  0.0093129705
## 26 -0.11614266 -0.0016904869  0.0105446406
## 27 -0.42852075 -0.0066491936  0.0208475965
```

```
dotplot(ranef(logMod, condVar=T))
```

```
## $VP
```

VP



Bonus Task

What is the predicted AT for Participant 10 in a task with 40 PAE?

Confidence Intervals for Random and Fixed Effects

```
confint(logMod)
```

```
##                2.5 %        97.5 %
## .sig01          0.290660357  0.4306436690
## .sig02          0.634333246  0.8185529828
## .sig03         -0.881780538 -0.8513384893
## .sig04          0.002868468  0.0056161554
## .sig05         -0.937868311 -0.7437788051
## .sig06          0.015246500  0.0214966570
## .sigma          0.228127802  0.2307459632
## (Intercept)    -0.265467275  0.0731725108
## abs_dev        -0.005695276 -0.0004631329
## log_devSc      0.008341204  0.0225499996
```

Plot Models with Data

```

for(loop in 1 : 27){

  plotF = function(x){(fixef(logMod)[1] + ranef(logMod)$VP[loop,1]) +
    (fixef(logMod)[2] + ranef(logMod)$VP[loop,2]) * (x) +
    (fixef(logMod)[3] + ranef(logMod)$VP[loop,3]) * 10*log(x)}

  if(loop == 1){
    curve(plotF,0,100,ylim=c(0,1),col="grey",
          xlab="PAE", ylab="AT")
  }else{

    curve(plotF,0,100,add=T,col="grey")

  }}

plotF = function(x){fixef(logMod)[1] +
  (fixef(logMod)[2] ) * (x) +
  (fixef(logMod)[3] ) * 10 * log(x)}

curve(plotF,0,100,add=T,col="black",lwd=2)

plotF = function(x){fixef(sqMod)[1] +
  (fixef(sqMod)[2] ) * (x) +
  (fixef(sqMod)[3] ) * (x)^2/100}

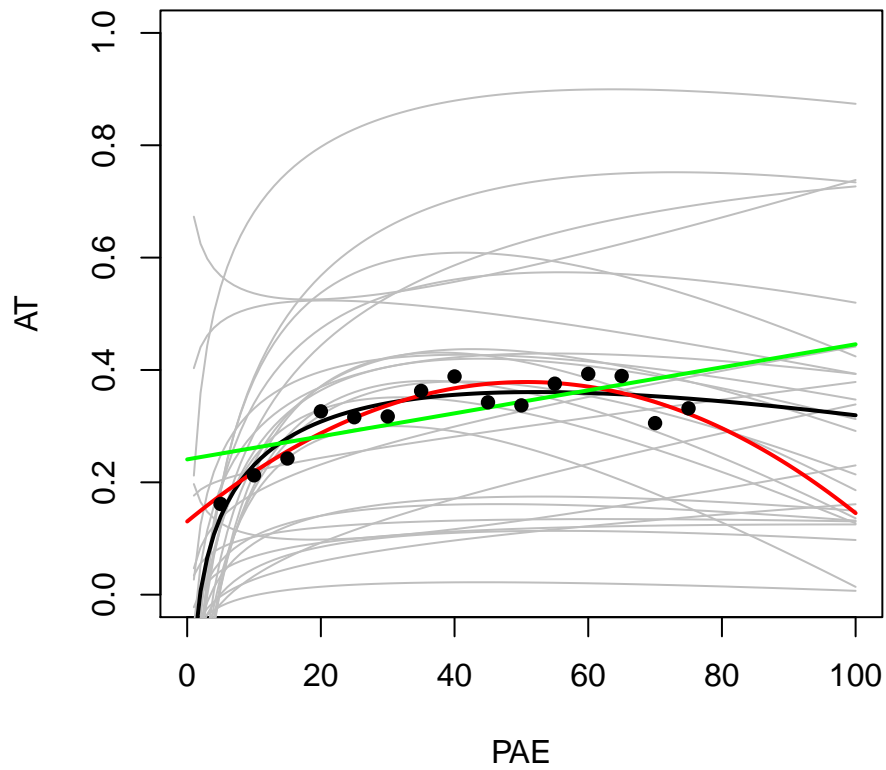
curve(plotF,0,100,add=T,lwd=2,col="red")

plotF = function(x){fixef(linearMod)[1] +
  (fixef(linearMod)[2] ) * (x) }

curve(plotF,0,100,add=T,lwd=2,col="green")

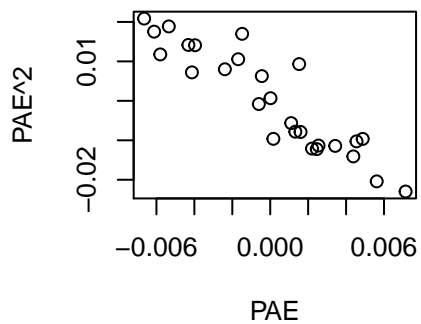
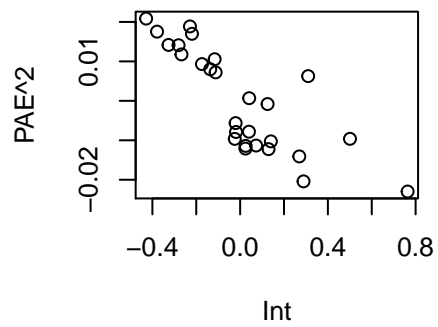
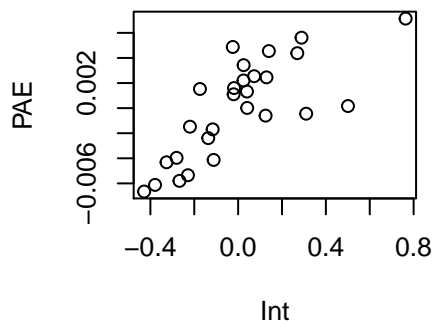
points(stepsManip,totalMeans,pch=16)

```



Scatterplot random effects.

```
layout(rbind(c(1,2),c(3,4)))
plot(ranef(logMod)$V[,1],ranef(logMod)$V[,2],xlab="Int",ylab="PAE")
plot(ranef(logMod)$V[,1],ranef(logMod)$V[,3],xlab="Int",ylab="PAE^2")
plot(ranef(logMod)$V[,2],ranef(logMod)$V[,3],xlab="PAE",ylab="PAE^2")
```



Visualizing Correlation of Random Slopes

```

plotF = function(x,a,b,c){fixef(logMod)[1]+a +
  (fixef(logMod)[2] +b) * (x) +
  (fixef(logMod)[3] +c) * 10 * log(x)}

curve(plotF(x,a=0,b=0,c=0),0,100,col="black",lwd=1,ylim=c(0,1),
  xlab="PAE",ylab="AT")

colPl=rainbow(121)

counter=1

for(loop in seq(-.006,.006,.0002)){

  curve(plotF(x,a=0,b=loop,c=-loop),0,100,add=T,col=colPl[counter])
  counter = counter + 1
}

```

